

Mart Ayı Sorusunun Cözümü

Öncelikle $1 + \frac{1}{k^2} + \frac{1}{(k+1)^2}$ ifadesini biraz düzenleyelim.

$$\begin{aligned}1 + \frac{1}{k^2} + \frac{1}{(k+1)^2} &= \frac{k^2(k+1)^2 + (k+1)^2 + k^2}{k^2(k+1)^2} \\&= \frac{k^2(k+1)^2 + k^2 + 2k + 1 + k^2}{k^2(k+1)^2} \\&= \frac{k^2(k+1)^2 + 2k^2 + 2k + 1}{k^2(k+1)^2} \\&= \frac{k^2(k+1)^2 + 2k(k+1) + 1}{k^2(k+1)^2} \\&= \frac{(k(k+1))^2 + 2k(k+1) + 1}{k^2(k+1)^2} \\&= \frac{(k(k+1) + 1)^2}{k^2(k+1)^2} \\&= \left(\frac{k(k+1) + 1}{k(k+1)} \right)^2 \\&= \left(1 + \frac{1}{k(k+1)} \right)^2 \\&= \left(1 + \frac{k+1-k}{k(k+1)} \right)^2 \\&= \left(1 + \frac{1}{k} - \frac{1}{k+1} \right)^2\end{aligned}$$

Bu durumda serimiz

$$\begin{aligned}\sum_{k=1}^n \left(1 + \frac{1}{k} - \frac{1}{k+1} \right) &= \left(1 + \frac{1}{1} - \frac{1}{2} \right) + \left(1 + \frac{1}{2} - \frac{1}{3} \right) + \dots + \left(1 + \frac{1}{n} - \frac{1}{n+1} \right) \\&= n + 1 - \frac{1}{n+1} \\&= \frac{n^2 + 2n}{n+1}\end{aligned}$$

olur.